Saffman Taylor Instability using a Hele Shaw Cell

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I. Overview

In this report, a picture is presented that demonstrates the Saffman Taylor instability using a Hele Shaw cell. First, a brief overview of hydrodynamic stability is provided, followed by a description of the instability phenomena. Then, information about how the experiment was conducted, followed by observations. Details of the photograph are followed by a critical analysis.

II. Introduction

Hydrodynamics stability is the study of stability of motions of fluids. Scientists such as Raleigh, Helmholtz, Kelvin and Reynolds in the 19^{th} century were amongst the first to investigate fluid stability rigorously [1]. The primarily purpose of hydrodynamic stability studies is to realize if a flow is unstable, and if unstable, to determine how it transitions to another flow. A flow is said to be stable if small disturbances initially in the flow die out or remain small, and is said to be unstable if these small disturbances grow. Fluid flows are subject to both stabilizing forces which tend to dampen out any disturbances, and destabilizing forces which promote the disturbances. The way in which these forces interact determines the nature of the resulting flows.

A classic example of flow instability is Rayleigh-Taylor (RT) instability (Rayleigh 1883; Taylor 1950) where a stratified fluid is accelerated from the lighter fluid to the heavier resulting in the formation of ligaments. The RT instability shows the imbalance between the stabilizing effect of buoyancy and the destabilizing effect of inertia. Saffman and Taylor [2] later recognized that the instability that arose at the moving interface between two incompressible, immiscible, viscous fluids in a porous medium was analogous to RT instability. The Saffman-Taylor (ST) instability may be demonstrated by the use of a Hele-Shaw setup, where a less viscous fluid is injected into a more viscous one constrained between two thin parallel surfaces. The resulting interface between the fluids exhibit wave like projections otherwise knows as “fingers”. In ST instability surface tension serves to stabilize the system by trying to minimize the surface area, while viscosity destabilizes the system promoting the growth of disturbances. The ST phenomena is observed in oil drilling applications, where the Carbon dioxide (less viscous fluid) is pumped into the ground (porous medium) to destabilize the oil-water (more viscous fluid) interface resulting in the oil coming out from adjacent wells in the form of “oil tongues”.

It has been shown that the wavelength of the initial fingers dependent on the film thickness and scales approximately as $\lambda \sim 4d$ where d is the film thickness and λ the wavelength. [3]. However the development of secondary fingers is more complex.

III. Experiment

Figure 1 is a schematic of the Hele Shaw setup we used to observe ST instability. It consists of a parallel plate arrangement, with a hole in the bottom plate though which the less viscous fluid is injected. The corn syrup is first poured on the bottom plate after which the top plate is gently lowered to prevent entrapment of air. The colored water is injected though the bottom hole by means of a syringe. A series of photographs was then taken as the fingers developed.

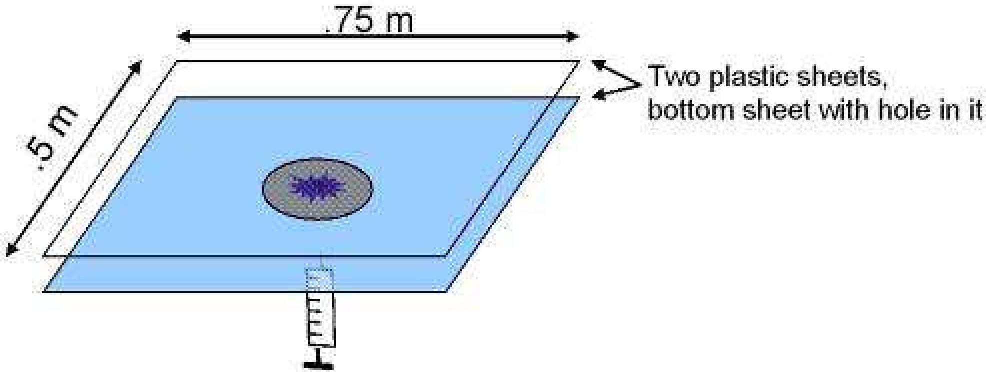

Figure 1. Schematic of the Hele Shaw setup

IV. Observations & Discussion

Initially the finger like projections appeared to develop from an approximate circular interface with the origin at the injection hole. However subsequent projections did not center on the injection hole, and initiated from other fingers as the disturbances now are initiated at different points on the interface.

Figure 2(a) Original Image

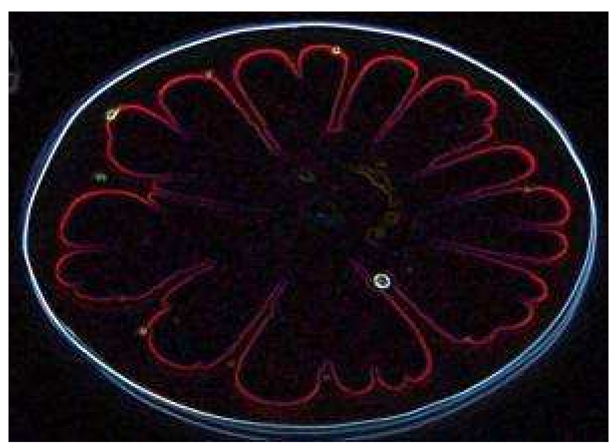

Figure 2(b) Image after manipulation

Based on stability theory, the growth rate and wave number of different modes is related to several parameters including the fluid properties and the velocity of the fluids. Regarding this, it was calculated that the ratio of densities of corn syrup to water to be about 1.5, while the ratio of the viscosities is about 75. From the dimensions of the syringe, injection hole and flow rate, we estimated that the average injection velocity is approximately 1 m/s. The film thickness was not measured since a spacer was not used to separate the plates. Thus we did not compare the wavelength to the film thickness.

Figure 2(a) shows the original (post-cropped) photograph taken on a 5 Mega pixel Digital Canon Power Shot S500. The field of view is estimated to be approximately .15 m wide by .1 m tall. The fluid was about 1 m from the camera. The focal length of the lens on the camera was 15 mm and the lens aperture was F/10. The shutter speed was 1/160 seconds. Using Adobe Photoshop 7.0, the image was manipulated first by cropping extraneous material (Figure 2(b)). Also, the image was then controlled with a "glowing edges" filter that allowed the extraction of the most valuable information of the photo to be shown in a beautiful manner. The glowing edges filter identifies the edges of color and adds a neon-like glow to them. The final image was 1270 x 903 pixels.

V. Conclusion

In future experimentation, it would be helpful to use a spacer between the two pieces of plastic in order to keep a constant distance. Non-Newtonian fluids could be used, which have been shown to result in different interesting patterns. Comparing the dimensions of the fingers from the experiment to that obtained from analytical expression would have provided more complete coverage to the project.

In conclusion, this photograph is not only beautiful, but also effectively shows the "fingers" associated with the classic Saffman Taylor instability problem.

VI. References

[1] P. G. Drazin, Introduction to Hydrodynamic Stability, Cambridge University Press, 2002
[2] P. G. Saffman, G. I. Taylor, The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid, Proc. Roy. Soc. London, Ser. A 245, 312, 1958
[3] L Paterson, Fingering with miscible fluids in a Hele Shaw cell, Phys. Fluids, 28 (1), 1985