Kelvin-Helmholtz Instability with Oil and Water

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I. Introduction

This image was created for first group project photo for the University of Colorado at Boulder Flow Visualization course. Although the set-up and build for the image was a group effort, the pictures and post-processing were individually done. The purpose of this image was to capture a phenomenon, usually seen in clouds, called the Kelvin-Helmholtz instability as well as further explore the world of flow visualization and photography.

II. Set-Up, Methods and Post-Processing

The inspiration for this shot came from some of the photos of the K-H instability in cloud formations. Some research revealed that it can be recreated with any 2 fluids of different viscosity and density. Some small scale "desktop" models exist currently on the market but in order to get a more detailed shot, the team decided to build a custom box larger than what is available. The build for this shot took a fair amount of construction. A clear acrylic box 22" wide, 7" tall and 3" deep was constructed for the shot. All the edges were machined to be smooth in order to create a water tight seal with the acrylic adhesive. The original dimensions called for the box to be about 40" wide but that was beyond the limits of the mill, therefore it was reduced to 22". After the edges were milled and the box was glued together, it was filled with approximately half water and half oil. The water was dyed blue using water based food coloring so it would not bleed color into the oil and to create more contrast between the fluids. Finally the box was cleaned with isopropyl alcohol to remove all smudges. The camera was mounted on a tripod and the box was tilted in front of the camera to capture the instability. It was found that bubbles quickly form and gather at the interface between the oil and the water. This creates some buoyancy effects that change the shear layer between the fluids, but the K-H instability was still elusively present. The camera was about 15 cm away from the fluid as seen in Figure 1.

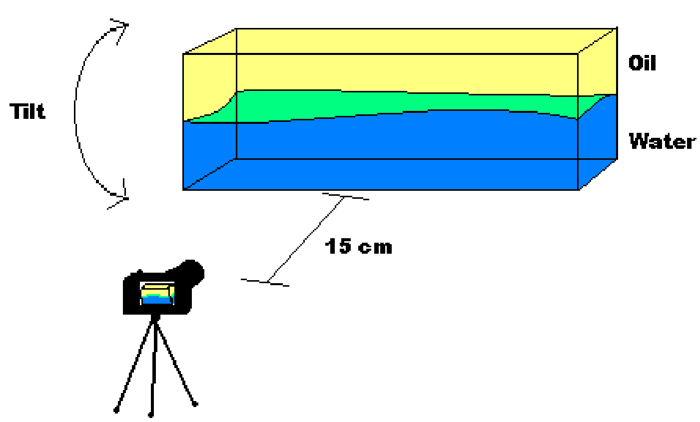

Figure 1- Set-Up Diagram

A shutter speed of 1/60 seconds was used with an ISO of 400 and an f-stop of 10. No flash was used and the only light was the florescent bulbs from above. In Photoshop, the saturation was bumped up and colors were adjusted to make the yellow and blue stand out more. The brightness was also increased slightly. Finally the photo was cropped to focus on the instability.

III. Analysis

The Kelvin-Helmholtz instability occurs when there is a shear force between varying density immiscible fluids creating different velocities within a continuous medium. Small perturbations cause the faster moving fluid, water, to amplify the turbulent flow into the wave that is seen in the image. [1] The perturbations in the oil pull kinetic energy from the water making small waves bigger and bigger, eventually curling up. This can be seen in Figure 2 where U_1 is the faster moving oil, and U_2 the slower water. The small perturbation occurring in step "b" grows larger and ultimately curls under creating the wave in step "e".

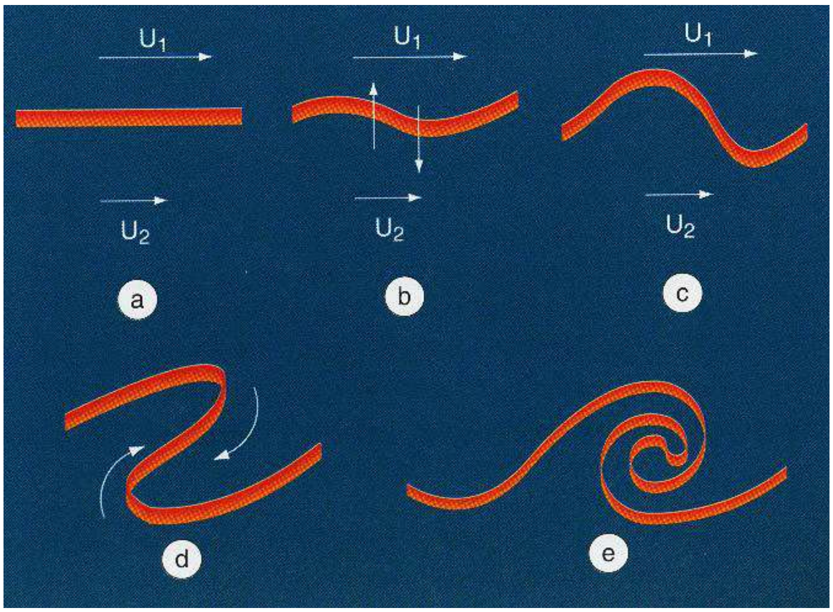

Figure 2- K-H Instability formation [2]

Before the box was tilted the fluids were in static stability. The less dense oil was floating on the more dense water. It is only from a stable system that the Kelvin-Helmholtz instability can be generated. Unstable system perturbations generally result in Rayleigh-Taylor instability.

The K-H instability occurs between any shear layers within a fluid, but can only be easily seen if the shear layer is between differently colored fluids. It is also seen in cloud formations when the dew point is such that the increased turbulence and pressure difference causes condensation to form and be visible against the translucent sky.

The derivation of the mathematical relationship begins with the 2D Navier-Stokes equations in each fluid. First Euler's equations are set up where *U* is the velocity vector, *x* is the position, *P* is pressure and ρ is the density. [2]

$$\frac{\partial U_j}{\partial x_j} = 0$$

$$\rho \frac{\partial U_i}{\partial t} = \rho\left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j}\right) = \frac{\partial P}{\partial x_i} + \rho F_i$$

Solution potentials, φ_1 and φ_2, are then found using associated velocities.

$$v_1 = grad\left[\frac{Ux}{2} + \Phi_1(x, y, t)\right]$$

$$v_2 = grad\left[-\frac{Ux}{2} + \Phi_2(x, y, t)\right]$$

The perturbations, ξ, are set equal to the fluid velocity component that is perpendicular to the interface.

$$[v_{1\perp}]_{y=\xi} = [v_{2\perp}]_{y=\xi} \cong \frac{\partial \xi}{\partial t}$$

The velocities are then projected onto the normal of the interface.

$$v_{iy} \cong \frac{\partial \xi}{\partial t} + v_{ix} \frac{\partial \xi}{\partial x}$$

Finally potentials, φ_1 and φ_2, can be solved in terms of the perturbations, ξ.

$$v_{1y} = \frac{\partial \Phi_1}{\partial y} = \frac{\partial \xi}{\partial t} + \frac{U}{2} \frac{\partial \xi}{\partial x}$$

$$v_{2y} = \frac{\partial \Phi_2}{\partial y} = \frac{\partial \xi}{\partial t} - \frac{U}{2} \frac{\partial \xi}{\partial x}$$

The equations above can be verified with mathematical fluid dynamics computer models and create fairly accurate representations of the phenomenon. [3]

IV. Conclusions

Overall, the experiment and image were successful. The bubbles in the image made it difficult to capture the K-H instability by itself, but also contributed to the originality of the image. The analysis revealed the complex nature of the seemingly simplistic phenomenon.

If I were to do this experiment again, I would change the dimension of the box in an attempt to limit the boundary disturbances and create a larger velocity difference between the fluids. I believe increasing the length of the box from 22” to over 40” would accomplish this. I would also try backlighting the box in an attempt to highlight the interface and depth of the flow.

The Kelvin-Helmholtz instability has been observed as long as the clouds have floated across the sky and more recent mathematical discoveries have led to a deeper understanding of the physics behind it. Even Van Gogh was inspired enough by its beauty to include it in his famous La Nuit Etoilee.

V. References

[1] Miami University, (2007). http://www.rsmas.miami.edu/users/isavelyev/GFD-2/KH-I.pdf

[2] Enseeiht Continuing Education, (2008). http://hmf.enseeiht.fr/travaux/CD0001/travaux/optmfn/hi/01pa/hyb72/kh/kh_theo.htm

[3] Anderson, John D. (1995), Computational Fluid Dynamics, The Basics With Applications.