Bubble through Corn Syrup and Water

Project 1: Get Wet
Flow Visualizations - MCEN 5228-10
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The complex behavior of bubbles rising through a fluid poses many physical questions that are not yet answered. Some of the better understood or documented physical relations for bubbles exist for air bubbles rising through water, which generally provide approximations for the flow characteristics of the bubble and attempt to describe the shape and motion the bubble may have. The aim of this project is to gain a better understanding of the flow phenomena that occur along a bubbles path through water by looking at a high speed image of a single bubble. The image produced shows complex disturbances of the bubble's surface and its surrounding environment. In order to understand the image, the forces that govern a single bubbles motion through the fluid and its shape, the disturbances that occur within the bubble and the bubble's affect on it surrounding environment will be investigated in the photograph.

In order to create the image, the flow apparatus needed to be able to generate a single bubble as well provide a method to differentiate fluid elements and show dynamic events. To do this, a glass container with flat walls was fitted with a flexible plastic tube at the bottom. Light corn syrup, dyed green, at the bottom of the container was used to help generate a single bubble and to provide a general idea of the bubbles affect on a fluid as it rose towards the surface. Water was then filled to the top of the container, leaving the denser corn syrup on the bottom and providing the medium for which the image could be easily viewed (see **Figure 1**).

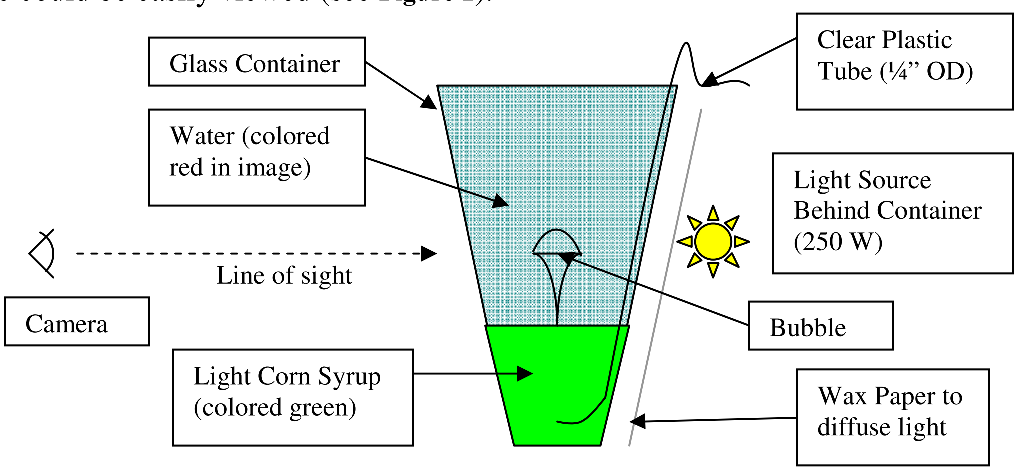


Figure 1: Schematic of the apparatus setup.

In this set up, as a bubble is created, the dense corn syrup forces any air blown through the tube into a single bubble. As the bubble breaks the interface between corn syrup and water, it begins to rise much quicker because water has a lower viscosity than corn syrup and thus resists the bubble's flow less. The photographed image is taken just a couple tenths of a second after this occurs. Because of the relatively large size of the photographed bubble (2" across by 11/16" high and a volume of approximately 0.79 in^2), it has already reached its maximum possible velocity in the image (determined by a balance of the opposing buoyant or lifting force and the drag force acting on the bubble which in this case are equal, creating no change in velocity; see Appendix)[4]. Rising at an estimated velocity of 11.42 in/s, the bubble has a Reynolds Number (Re) of 7550, an Eötvös Number (Eo) of 115, and a Morton Number (Mo) of 4.14×10^{-11} based on the waters fluid properties. These non-dimensional numbers, primarily Eo and Mo, are used to describe a bubbles shape as it travels through a fluid as well as other fluid characteristics (**Figure 2** shows where the photographed bubble falls on a Eötvös/Morton curve and the expected bubble shape – spherical-capped bubble).

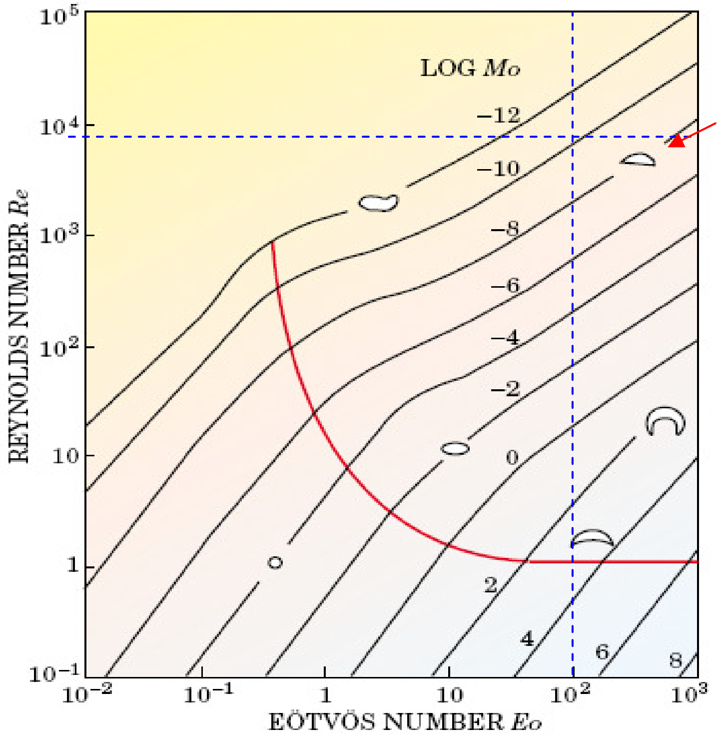


Figure 2: Graph showing relationship of Eötvös, Morton and Reynolds numbers and the shape of a bubble. Figure from Lohse[3]

One of the more striking characteristics of the image is the ripples visible along the bubbles surface. These ripples are caused by an interaction between the surface tension of the water and the shear stresses (cause by tangential forces) on the bubbles surface. The surface tension of the water around the bubble is constant in magnitude at the air water interface of the bubble. The shear stresses along the surface of the bubble vary depending on the local velocity of the water passing across the air surface due to the momentum of the passing fluid. For small bubbles, the surface tension is enough to overcome these minor variances in viscous shear along the bubble. However, for larger bubbles such as the one in the photograph, larger fluid velocity, due to high buoyant forces and a large surface area, create greater variances in local shear. Because surface tension is constant in magnitude, the variances in shear create a slight instability in the bubbles surface that becomes compounded, resulting in ripples along the bubbles surface. At high enough velocities, the shear along the bubbles surface would be able to break the bubble apart, however, in this case velocity is limited due to drag and the bubble maintains the relatively constant general shape known as a spherical-cap bubble.[1]

In order to create the bubble image, two flow visualization techniques were utilized: light bending and boundary marking. Light bending is the primary technique used as this is what makes the bubble visible. Light is shined though and reflected off of the bubble to create texture in the image and show the bubbles shape. This technique utilizes the difference of refractive index of water (η≈1.3) and air (η≈1). Boundary marking was used to show the dynamic effects of the bubble traveling through the water.

This was done by coloring corn syrup green. The green corn syrup is pulled behind the bubble because there is a section of low pressure left in the bubbles wake. Furthermore, some of the green corn syrup that originally traveled upwards with the bubble slides off the top of the bubble providing additional dynamic information of the fluids motion around the bubble. There is also light bending technique associated with the corn syrup as it has a larger index of refraction than the surrounding water. In order to create the lighting effect in the image a 250W tungsten light bulb was placed directly behind the glass container, taking advantage of the forward scattering of light. Diffuse natural light was also used to ensure that the camera would receive enough light for the relatively fast exposure time.

The bubble photograph was created using a Nikon D50, digital SLR body with an image resolution of 3008x2000 pixels. The field of view of the image is 4.25"x6.5" giving a spatial resolution of 4.51×10^{-6} in^2/pixel (length of a pixel is 0.0212") and was taken approximately 7.5 ft from the lens. The lens focal length used was 300mm on a Nikon EWD 70-300mm 1:4-5.6D telephoto lens (by Nikon Corporation) with a UV filter. The image was exposed at an f-stop of 5.6 and a shutter speed of 1/640sec. The velocity of the flow was 11.42 in/s, therefore the bubble moved 0.0178" during the exposure which is less then the spatial resolution in terms of the length of a pixel. There was minimal additional processing to the image using Photoshop; the image was first cropped from 3008x2000 pixels down to 2644x1465 pixels to remove unnecessary information and the dodge tool was used to brighten the bubble surface (using highlights range).

This image reveals information about a bubbles shape as well as some of the dynamics of the surrounding fluid. The corn syrup falling off of the top and sides of the bubble and the plume that follows the bubble help to better understand the bubbles effect on its surrounding environment. Furthermore, the high speed image shows rippling on the bubbles surface helping us gain a better understanding of the nature of the bubble itself. Some of my personal favorite aspects of this image are the texture created on the bubbles surface, the balance between light and dark created by the high intensity light and the repetition of the dark shadows on the top of the bubble and on the top of the container. I felt that the physics of this the bubbles motion were shown very clearly, however, the corn syrup added a level of complexity to the image that likely changed some of the physics compared to a pure water scenario. Some questions I was not able to answer while doing this project were: is there a relationship between the acoustic properties of bubbles and the ripples seen on the bubbles surface? Is there a better way to generate an image that will display the fluid motions about the bubble (ie. different complimenting liquid, different lighting, different angle)? Overall, I did fulfill my goal of learning how to use a camera, creating an image that I felt strongly showed multiple physical phenomena, some of which I was not expecting, and created something that was visually appealing. The corn syrup was a last minute change to my set up that I felt added an additional level of texture to the image that I was especially happy with, however, I would have liked to have found another fluid that would have had a lesser effect on the fluid interaction between bubble and water. To develop this idea further, I think it would be interesting to experiment with a number of different fluids and compare the physics between each medium. A higher level of control over the volume of air introduced into the fluid would also be helpful as it would be interesting to see the relationship of bubble size to the Eötvös/Morton model for determining bubble shape.

Appendix

Calculations of fluid characteristics and dimensionless parameters

In all calculations the air bubble is assumed to be affected by the water only, thus affects from the corn syrup are neglected.

For the bubble in the photo, the following approximations were used to determine the characteristics and dimensionless parameters for the bubble:

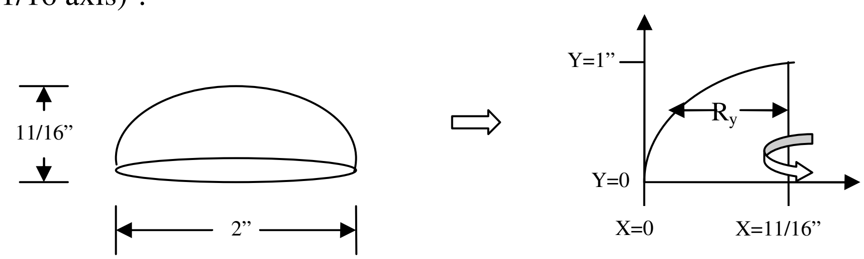
Velocity of the bubble through the water:

t = 0.70 sec
D = 8 in
U = D/t = 11.43 in/s

Where t is the time it took for the bubble to travel a distance D through the water, providing U, the average velocity of the bubble (note: t was calculated over 11 trials to obtain a good approximation for the transit time).

Volume of the bubble:

The volume V of the bubble was determined by approximating the volume of a circular paraboloid with similar dimensions as the bubble. A 2D equation can then made to represent the paraboloid, thus allowing integration using the cylindrical shell method in which the volume of a curve is determined by revolving the curve about an axis (in this case the x=11/16 axis)[5].



Given the bubble dimensions of 2” diameter and 11/16” height, the following equation was generated to evaluate the volume of the bubble:

$$y = \sqrt{16/11} * \sqrt{x} \Rightarrow$$

$$\Rightarrow R(y) = 11/16 - (11/16)y^2 = 11/16 * \left(1 - y^2\right)$$

$$V = \int_0^1 \pi\left[11/16 * \left(1 - y^2\right)\right]^2 dy = \left(\frac{11}{16}\right)^2 \pi * \left(y - \frac{2y^3}{3} + \frac{y^5}{5}\right)_0^1 = 0.79 in^3$$

The equivalent spherical diameter of the bubble can be used to determine non-dimensional numbers that can be used to gain a better understanding of the fluid interaction and to help categorize the geometry of the bubble[1]. The equivalent spherical diameter is thus

$$d = \left(6 * V / \pi\right)^{1/3} = (6 * 0.79 / \pi)^{1/3} = 1.15 in$$

Non-dimensional parameters:

Using the equivalent spherical diameter and the fluid properties of water, the following non-dimensional parameters may be determined.

Fluid properties of water at room temperature (68°F):

Kinematic viscosity: ν = 1.12x10^{-6} m^2/s = 0.001736 in^2/s;

Surface tension: σ = 0.072 kg/s^2 = 0.1587lbm/s^2;

Density: ρ = 1000 kg/m^3 = 0.0361 lbm/in^3

Specific Gravity: γ = 62.4 lb/ft^3

Reynolds Number:

$$\text{Re} = \frac{Ud}{\nu} = \frac{(11.43in/s)(1.15in)}{0.001736in^2/s} = 7550$$

Eötvös number[1]:

$$Eo = \frac{\rho d^2 g}{\sigma} = \frac{(0.03613lbm/in^3)(1.15in)^2(386.4in/s^2)}{0.1587lbm/s^2} = 116 \cong 115$$

Morton number[1]:

$$Mo = \frac{g\nu^4\rho^3}{\sigma^3} = \frac{(386.4in/s^2)(0.001736in^2/s)^4(0.03613lbm/in^3)^3}{(0.1587lbm/s^2)^3} = 4.14\times10^{-11}$$

Weber number[1]:

$$We = \frac{\rho U^2 d}{\sigma} = \frac{(0.03613lbm/in^3)(11.43in/s)^2(1.15in)}{0.1587lbm/s^2} = 34.1$$

Drag Coefficient[1]:

$$C_d = \frac{4dg}{3U^2} = \frac{4*(1.15in)(386.4in/s^2)}{3*(11.43in/s)^2} = 4.52 \cong 4.5$$

Using the drag coefficient, the *drag force*, F_d, can be determined by the following equation,

$$F_d = -0.5*\rho U^2 A C_d$$

$$A = \pi d^2/4 = \pi(1.15)^2/4 = 0.1.03in^2$$

$$F_d = -0.5*(0.03613lbm/in^3)(11.43in/s)^2(1.03in^2)*4.52*(\frac{slug}{32.2lbm})*(\frac{ft}{12in})$$

$$F_d = -0.0285lbf$$

where A is the cross sectional area of the bubble based on the equivalent spherical diameter.

The *buoyant force*, F_B, can be determined by:

$$F_B = \gamma V = (62.4lbf/ft^3)*(0.79in^3)*(\frac{ft}{12in})^3 = 0.0285lbf$$

Therefore, the buoyant and drag forces balance and the bubble is traveling at maximum velocity (no longer accelerating). Note: because the mass of the air in the bubble in very small it is neglected for this calculation.

References:

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4. Munson, Bruce R., Donald F. Young, and Theodore H. Okiishi. Fundamentals of Fluid Mechanics. 5th ed. Hoboken: John Wiley & Sons, Inc., 2006. 69, 520.

5. Thomas, Jr, George B., and Ross L. Finney. Calculus. 9th ed. New York: Addison-Wesley Co., 1998. 379-380.