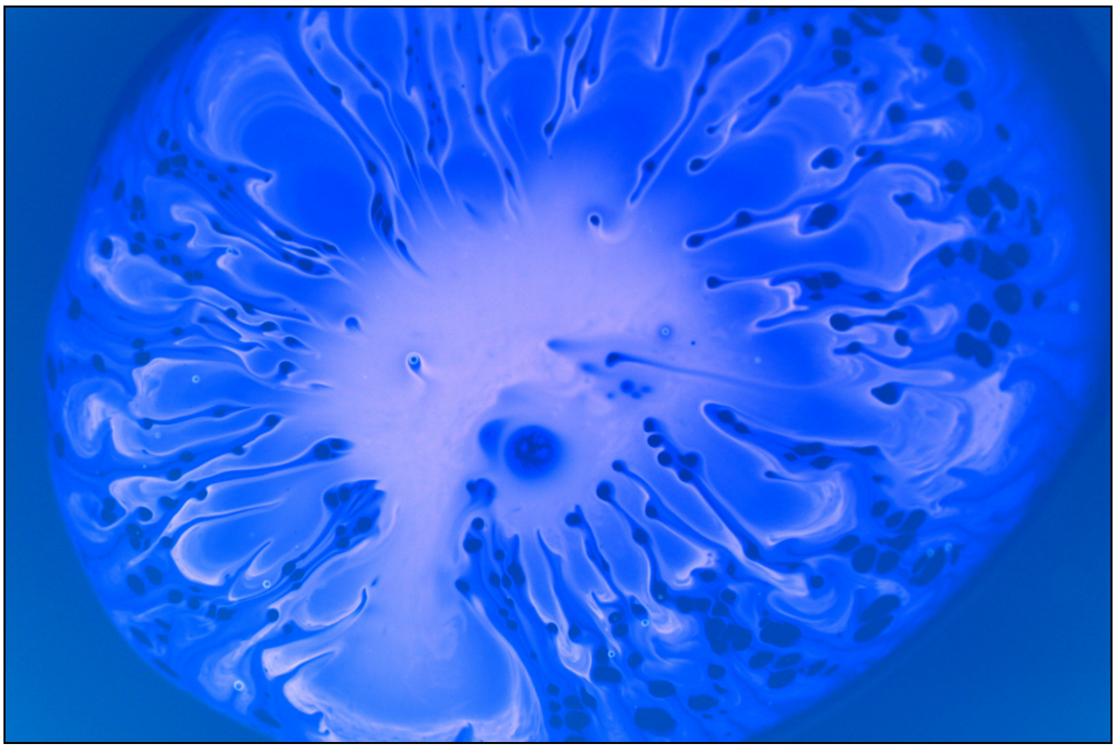
Saffman Taylor Instability



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Flow Visualization

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Project #3 (Team Beta)

Due 10/25/07

Purpose:

This is the third project of the semester, and our group played around with visualizing the Saffman Taylor Instability. There are several papers and experiments that have been performed regarding this instability, and so the main objective for our group wasn't to show the same thing everyone else has been showing in the past, but to show some variations. The main intent specific to this paper is to show a variation of the Saffman Taylor Instability where the fluid diffuses rather quickly inside the more viscous fluid. The photograph captures this variation, and the physics can be explained somewhat fundamentally, although other things may be more complex. Our group was originally going to attempt creating an artificial landscape by injecting dye droplets in vegetable oil, but we found that it was extremely difficult to control.

Experiment Setup:

To capture this image, the Hele Shaw Cell was used from the Media Shack in the ITLL. It wasn't actually used as a cell, but just to give a legitimate visualization for the picture. So, the only thing that was performed differently was the fact that the top glass cover was not used to essentially create a 2d Flow. The attempted creation of flow was performed by injecting hydrogen peroxide diluted green food coloring dye directly into the middle of a circular corn syrup layer on the free surface of the acrylic. A sketch of the experimental apparatus is shown below in figure 1. The dilution of the hydrogen peroxide and green dye was roughly half and half.

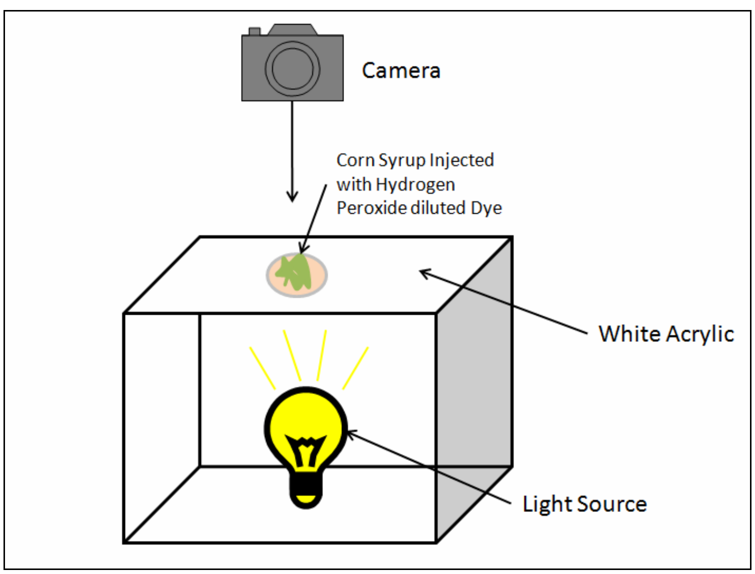

Figure 1: Experimental Setup

Physics:

The main idea behind the Saffman-Taylor instability is when a fluid of much less viscosity is introduced into a fluid with much higher viscosity, creating a finger-like pattern as it moves through the higher viscosity fluid. This happens solely by the existence of a pressure gradient, which is the driving mechanism for the movement of the less viscous fluid. It is shown in the book *Fluid Dynamics for Physicists* [1], that the pressure gradients of the two fluids can be given by:

$$\frac{\partial p'}{\partial x} = -\frac{12\eta' U}{d^2} \quad , \quad \frac{\partial p}{\partial x} = -\frac{12\eta U}{d^2}$$

The thickness of the corn syrup layer is d, p' and p are the pressures of the hydrogen peroxide diluted food coloring and the corn syrup respectively, +x is the direction that the hydrogen peroxide is moving, η' and η are the respective viscosities, and U is the uniform velocity that the hydrogen peroxide is moving at. If we assume that the curvature between the interface of the hydrogen peroxide and the corn syrup is constant, then we can write:

$$p' = -\frac{12\eta' U}{d^2}(x - Ut) + p_0 \quad , \quad p = -\frac{12\eta U}{d^2}(x - Ut) + p_0$$

Time is given by t, and p_0 is the initial pressure. Before going into more complex details about this instability, we can pick out terms, and determine roughly how big p' and p are relative to each other. The viscosity of the hydrogen peroxide is roughly the same as water, or 1.245cp, and the viscosity of Karo corn syrup is roughly 2000-3000 times larger than that. This causes p to be much larger in magnitude than p', as one can see from the above relations, and is the driving mechanism for the displacement of the corn syrup by the hydrogen peroxide. Looking at the picture specific to this experiment, the formation of the fingers does not seem to be completely stable, and there are effects that are different from expected behavior. One hypothesis for these somewhat "jagged" finger-like formations lies in the injection of the hydrogen peroxide into the corn syrup. Since there was an initial velocity outward when the injection was performed, this may have caused U to be a lot larger than normal. Another guess for the jagged-like behavior is the stability of the hydrogen peroxide itself. The acidic properties of the liquid may have caused it to spread differently than water would have. Either way, the expected smooth finger-like behavior wasn't apparent in this particular experiment, which successfully completes the intent of showing a different variation of this instability.

Photographic Technique:

The temporal resolution did not matter in this photograph, because it was taken when the fluid was practically static (It had evolved already). This photo was also taken up really close, and so a macro lens was used to give ideal focus. The light below the white acrylic sheet provided nice illumination for the experiment, because it was evenly dispersed through the sheet, and there was no glare affecting the outcome.

- Make/Model: Canon EOS Digital Rebel   2048 X 1360 Pixels
- X res: 180 lines/inch   Y res: 180 lines/inch
- Actual specs for photo: 50mm focal length, 4.0 F-Stop
- Field of View: ~ 3 inches
- Exposure Specs: 1/4000 sec exposure, aperture value of 4.0, ISO speed rating of 400

Because there was no information lost, the picture was inverted in Adobe Photoshop to make it more artistically interesting. The original is artistically legitimate, but inverting it makes it somewhat aesthetically mysterious, which is a nice touch. Other Photoshop adjustments include some contrast and brightness touchups. Below is a pre-Photoshop picture (figure 2).

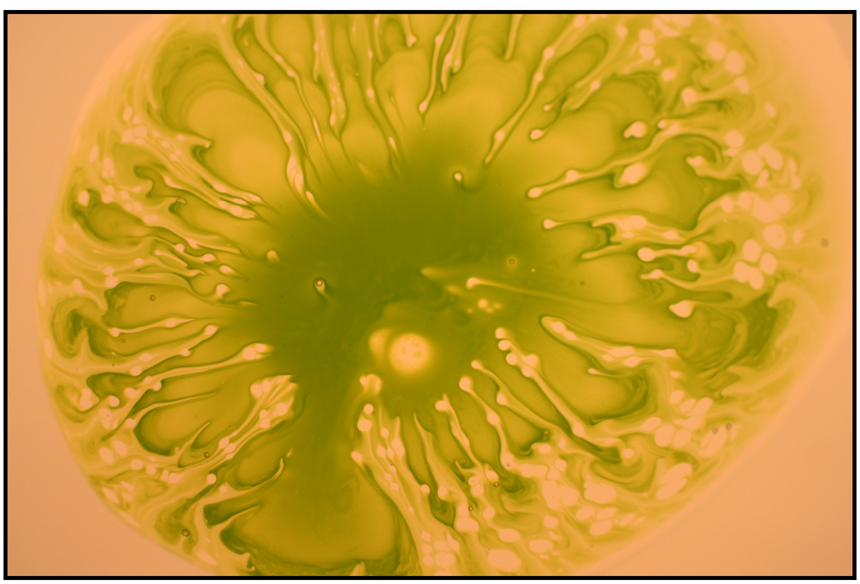

Figure 2: Pre-Photoshop (Original) Photograph

The Image:

The image is moderately intense, and the science is shown wonderfully. The Saffman Taylor fingers are shown, but more violently than normal. The inversion of the picture makes it more mysterious, and gives it some nice contrast. The experiment could have been planned better, and in the future it would be nice to have everything set in stone, so that our group will not have any problems. The series of photos that were shot actually turned out pretty decently, considering the confusing circumstances and misunderstandings of the requirements for the final report.

References:

[1] Faber T.E. (1995). Fluid Dynamics for Physicists. New York: Cambridge University Press.