

Flow Visualization  
MCEN 5151

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**A non-Cylindrical Stream Showing Signs of the Rayleigh instability.**



**Purpose:** The main object of this photo is to capture a clear image of a non-cylindrical fluid stream as it begins to experience the Plateau-Rayleigh instability. This particular image shows JP-8 Diesel fuel 1 inch downstream after being pushed through a slit nozzle. In this case the fluid stream shall, from now on be called the fluid “sheet.” This is because the fluid appears sheet like directly after exiting the slit nozzle. The image compliments the descriptions and predictions of the Rayleigh-Taylor instability. Furthermore it gives insight as to how to apply these theories to non-cylindrical fluid streams.

**Flow Apparatus:** The flow apparatus consists of two main parts, a pressurized cylinder and a slit nozzle. This specific setup creates the parameters of a theoretical fuel injector. The pressurized cylinder creates a line pressure of 2.5psi. The slit nozzle is  $\frac{1}{4}$  inch long by 4 thousandths thick. The combination creates a flow rate similar to that which is necessary to power an engine. Nozzle geometry and initial fluid sheet is shown in figure 1.

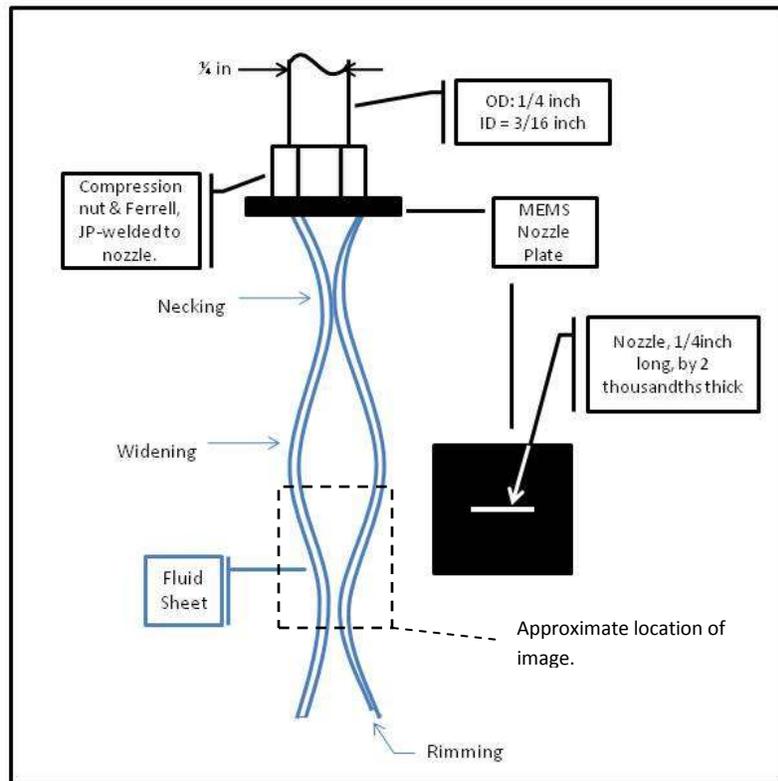


Figure 1, Flow Apparatus & fuel flow.

**Flow:** As shown in figures 1 and 2 the flow sheet necks and widens. This is a result of the effects of very small perturbations of the fluid flow. These small perturbations are amplified by the surface tension of the fluid. The surface tension can be thought of as a force acting to minimize the surface area of the fluid.

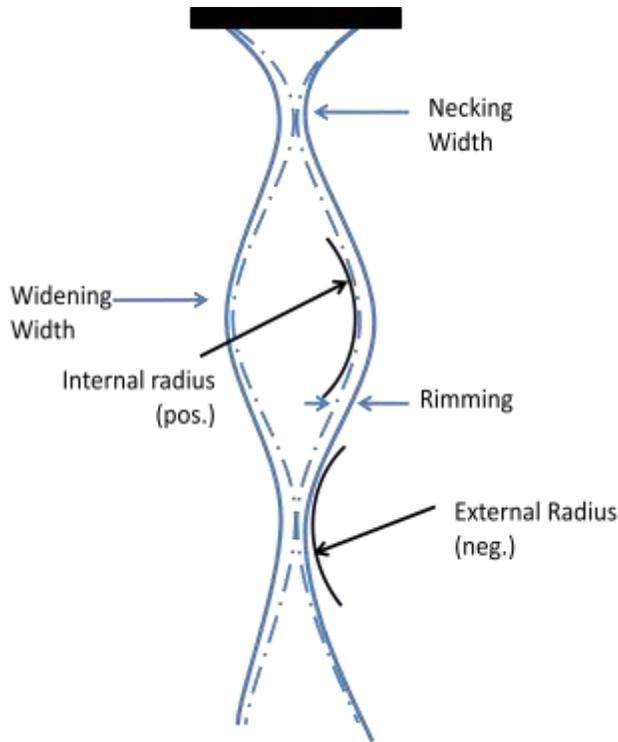


Figure 2, Initial Flow Sheet

The thinning and thickening shown in the figures is common with all falling fluid streams. It is considered the capillary instability and it is caused by a pressure difference described by the Young-Laplace equation. [1]

$$\sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right) = \Delta P$$

Where  $\sigma$  = surface tension,  $r_1$  = 1<sup>st</sup> principle radius,  $r_2$  = 2<sup>nd</sup> principle axis, &  $\Delta P$  = pressure differential between the two phases. Note that this equation is for cylindrical flow not flat flow. [1] However, the theory can be applied to the flat flow.

Notice in figure 2 that there are four phases. Or, instead think of them as pairs: necking and widening as one, and internal and external radii as two. Take the first pair. Necking corresponds to a small radius and the widening corresponds to a large radius. Surface tension laws govern that smaller stream has a higher net surface tension than a wide stream. Applying that to the flat

sheet flow seen in figure 2 the stream would want to pinch at the necking points. However the second pair tries to balance the first pair. Notice the internal radius vs. the external radius. Surface tension laws further state that a concave fluid surface relative to the inside of the fluid has a surface tension that squeezes, whereas a convex surface, relative to the inside of the fluid, pulls the fluid outward. The two pairs do indeed counter balance each other for the initial time the stream is formed. [1] However, with all streams eventually one of three things will happen: [2]

1. If the fluid is slow and non-viscous it will allow enough time to pass for the fluid stream to pinch off into droplets. To determine where the stream will pinch off one needs to consider the wave number,  $k = \text{peaks (or trough's)}/in$ . Note peaks correspond to widening and troughs correspond to necking. Whichever opposing radius times the wave number from the different pairs is closer to 0.7 their corresponding surface tension effects will over power the other. Remember though the stream's wave number is not constant throughout the length of the stream. This is one of the reasons why droplets vary in size. [2]
2. The fluid flow is falling faster than the acceleration of gravity. Small instabilities in the surface of the fluid rapidly form clumps of fluid. Given enough time to propagate, these clumps form

Rayleigh-Taylor figures. They are called that because they look like figures protruding from the surface of the fluid. [3]

3. If the fluid is moving fast inertial effects on the boundary layers between the fluid and surrounding fluid can start to affect the fluid stream before the surface tension does. When this happens the fluid forms liquid ligaments. This phenomenon is described as the Kelvin-Helmholtz instability. [2]

**Liquid Figures:** Simply put, liquid figures are a result of a fluid falling faster than the acceleration of gravity. When a fluid undergoes such a motion the air below it cannot supply enough upward pressure to the falling fluid to retain its shape. Theory states that the radius would increase. However, the upper limit as to how much the radius will expand is limited by the surface tension of the fluid. Furthermore there are instability in its surface of the fluid which quickly form dimples. Soon the fluid will form clumps or gathering which look like finger protruding from the fluid. [3]

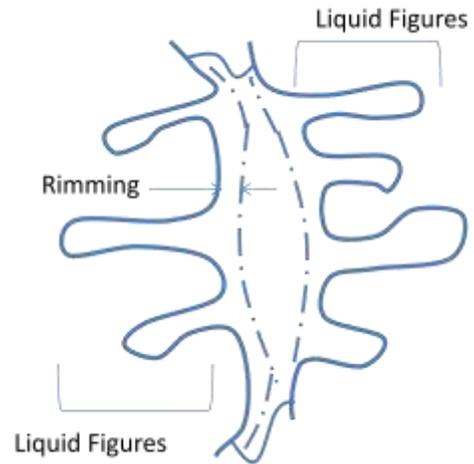


Figure 3, Downstream flow showing Rayleigh-Taylor figures.

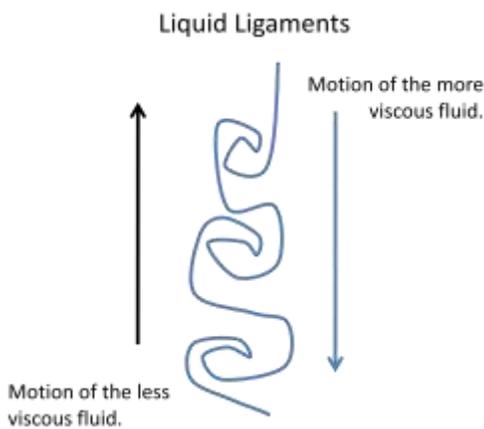


Figure 4, A profile of possible Liquid Ligaments.

**Liquid Ligaments:** Liquid Ligaments are observed when a fast moving fluid is passing through a slow moving fluid. An artist's rendition of the ligaments is shown in figure 4 note that liquid ligaments are much smaller than the liquid figures. In fact liquid ligaments often form on liquid fingers. This can be thought of as the effect of the shear forces acting on the stream.[2]

The shear instability arises from the vortex boundary layers of the two fluids. In figure 4 there are two boundary layers:  $\delta$

corresponding to the gas' boundary layer, and  $\delta_L$

corresponding to the liquid boundary layer. A higher viscosity results in a smaller vortex boundary layer. Note that the viscosity of a liquid is in fact much higher than that of a gas. This fact results in an assumption that  $\delta_L$  is negligible compared to  $\delta$  and thus  $\delta_L=0$ . It

is the vortices found in the gas's boundary layer that pull out the liquid out in the form of a spiral cortex. [2]

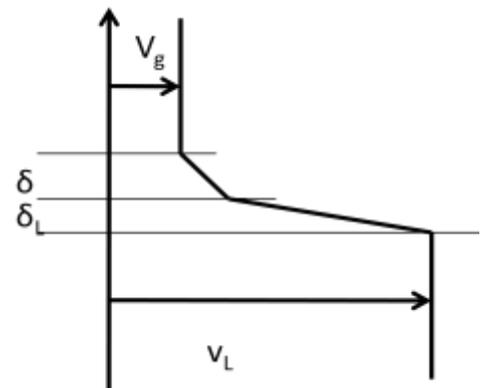


Figure 5, The piecewise linear velocity profiles of a typical inflectional instability

**Which instability over comes the other:** There are several fluid parameters which govern which instability will dominate the flow. Most of relevance parameters, specific to to the photo, are all captured with the Weber Number.

The Weber number for a jet is defined as the *Weber number*  $= \rho_L v^2 h_o / \sigma$ . Where  $\rho_L$  is the density of the liquid,  $v$  = the initial velocity of the stream,  $h_o$  = the characteristic diameter of the stream, and  $\sigma$  = the surface tension of the stream. [2]

Given the following data of the geometry of the flow:

$$\rho_L = .775 \text{ kg/L [4]}$$

$$v = 6.628 \text{ m/s}$$

$$h_o = \frac{4 * \text{Area}}{\text{Wetted Perimeter}} = 4 * 1/4" * 0.002" / (1/4" + 1/4" + .002" + .002") = 9.921 \times 10^{-4} \text{ in}$$

$$\sigma = 0.016 \text{ N/m [4]}$$

$$\text{Weber number} = \rho_L v^2 h_o / \sigma = 214 \text{ [2]}$$

Values of the Weber number corresponding to when the shear effects dominate are when the

$$\text{Weber Number} > \left( \frac{\rho_l}{\rho_g} \right)^2 = \left( \frac{0.775 \frac{\text{kg}}{\text{L}}}{1.055 \times 10^{-3} \frac{\text{kg}}{\text{L}}} \right)^2 = 735 \text{ . [2]}$$

Notice the calculated Weber number is not in the region where shear effects dominate and the Weber number is not < 1. This implies that the Plateau-Rayleigh and the Kelvin-Helmholtz instabilities must be dominated by the Rayleigh-Taylor instability. [2] If the Plateau-Rayleigh instability was dominating the sheet would be broken into droplets. If the Kelvin-Helmholtz instability dominated the flow the surface of the fluid would be rough with the liquid ligaments. Comparing this theory to the image which shows a smooth intact sheet, it becomes apparent that the Rayleigh-Taylor instability.

**Visualization Technique:** To photograph this image an SLR Camera was used. The nozzle was positioned 1 inch above the field of view of the camera. All lights were turned off in the room. A flash lamp was positioned directly behind the fluid sheet at the top of the image. The flash lamp had a flash duration of 2 micro-seconds. This allows the imaging of fast moving flow with little to no motion blur. The camera's aperture was wide open to allow in as much light in as possible light. A thin piece of blue tissue paper was suspended an inch in front of the flash lamp. (i.e. the tissue paper was suspended in between the light and fluid sheet.) This gave a nice smooth blue background. The camera had a slow shutter speed which allowed me to manually trigger the flash lamp while the shutter was open.

### Camera & Photograph Details:

#### Camera Details

Camera	Nikon D70's
Sutter Speed	10/60 sec
F-Stop	f/5.4
Lens	105 mm 1:2.8 DG MACRO
Focal Length	~4inch

#### Photograph Details

	Original	Final Edit
Field of View	1"x2/3"	1/2"x2/3"
Number of Pixels	3008x2000	1600x2000

Photoshop Process: The image was imported in AdobePhoto Shop and enhanced. The image was cropped, allowing the total focus of the image to be of the fluid. In the RGB scheme the color blue was made more vibrant i.e. the darker blue pixels were made lighter and brighter. This creates a better profile and nicer definition of the between the fluid and the background. Figure 6 shows the image pre and post the digital edit.



Figure 6, The image before and after the digital enhancement.

**Image:** The image shows a sheet of fluid one inch down from a slit nozzle. It is important to note that the fluid sheet, at the top of the image, is where it is widening and about halfway down the image it

the sheet necks. It is obvious, because of the presence of the liquid figures that the Rayleigh-Taylor instability dominates over the capillary or shear instabilities. The characteristic length is a good approximation for estimating fluid flow properties of non cylindrical flow. But since the flow is bi-symmetric numerical analysis of such flow is complicated. Further investigation needs to be done specifically into the apparent rimming effect. Notice in the image where it is wider the darker rims. These darker rims correspond to more fluid presence. A cross section of the flow may look like figure 7. This rimming effect is believed to be another effect of the capillary instability, where again there are the two pairs of opposing surface tension parameters. The Plateau-Rayleigh equation could define the formation of this rim.

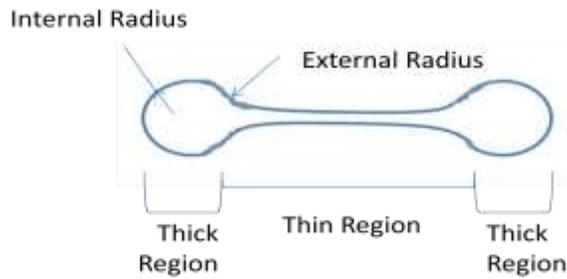


Figure 7, Cross section of initial stream. Thick regions correspond to rimming regions.

**Citations:**

[1] B. Gun, "Proof of the Young-Laplace equation using the theory of calculus of variations applied to petroleum fluids," *Petroleum Science and Technology*, v 21, n 7-8, p 1159-1165 (2003)

[2] J. Eggers and E. Villermaux, "Physics of liquid jets," *Reports on Progress in Physics* 71, p79-180 (2008)

[3] D. Shapr, "An Overview of Rayleigh-Taylor Instability," *Physica* 12D 2-18 (1984)

[4] J. Risher, P. Bittner, & S. Rhodes, "Toxicological Profile for JP-5 and JP-8," U.S. Department of Health and Human Services, Public Heal Service, Agency for Toxic Subastances and Disease Registry (1998)