Flow Visualization – First Image Assignment

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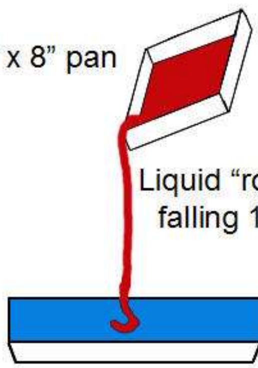
The intent of this image is to illustrate a coiling instability in a shear-thickening non-Newtonian fluid. Both the blue and red fluids are suspensions of corn starch in water. Originally I had hoped to photograph an impact between a hammer and the suspension. Unfortunately the viscosity of the fluid increased to such a degree that no significant ripples were visible on the surface of the fluid, and the effect was quite anticlimactic. Pouring the suspension, on the other hand, produced a coiling instability and showed the shear-thickening behavior of the mixture.

The suspension was divided between two baking pans, an 8" x 8" and a 9" x 13", with roughly two thirds of the fluid in the larger pan. The red mixture in the smaller pan was poured into the pan of blue suspension from a height of 12 inches. This produced a liquid "rope" approximately 0.1 inches in diameter which underwent a coiling instability.



Suspensions of corn starch in water are non-Newtonian shear-thickening fluids (Fall, et all 2008). These suspensions have a yield stress on the order of 0.3 Pa, and behave as solids when still (Fall, et all 2008). Above the yield stress, the mixture behaves as a liquid whose viscosity decreases as shear rate increases, until reaching a critical shear rate, at which point the viscosity increases abruptly until the suspension again behaves as a solid (Fall, et al 2008). This behavior is caused by a reentrant jamming transition, in which particle collisions within the suspension at higher stresses cause the viscosity to increase (Fall, et al 2008). Initially, shear stress allows the suspended corn starch particles to roll over each other, but at higher shear stresses the particles "jam" (Fall, et al 2008).

The pronounced shear thickening behavior of the fluid, combined with its high viscosity, allows the coiling instability to be observed. When the falling suspension impacts the stationary fluid, stresses are high, and the viscosity in both becomes very large. This behavior causes the "rope" that has recently landed to remain very well defined. As more "rope" falls, the older sections are under less stress, and the viscosity decreases. This allows the "rope" to coalesce with the suspension in the lower pan. The image shows the transition from high to low viscosity clearly. The "rope" is well defined on the left, and coalesces fairly abruptly on the right. If it were a high viscosity Newtonian fluid, the coil would join the standing fluid gradually and steadily, instead of the fairly abrupt coalescence observed here. The suddenness is caused by the change in the viscosity of the liquid.

Coiling instabilities can be divided into three groups: viscous, inertial, and gravitational (Maleki, et al 2004). A fluid undergoing a coiling instability experiences viscous, inertial, and gravitational forces, and the relative magnitudes of these forces determine which regime it falls into. When viscous forces dominate, viscous coiling occurs (Maleki, et al 2004). Gravitational coiling occurs when the viscous and gravitational forces are approximately equal with minimal inertial forces, and inertial coiling takes place when inertial and viscous forces are approximately equal with negligible gravitational forces (Maleki, et al 2004).

The relative influence of these forces is shown by the Reynolds number (Re) and the Froude number (Fr) (Munson et al 2009). The Reynolds number is the ratio between the inertial and viscous forces, while the Froude number is the ratio between the inertial and gravitational forces (Munson et al 2009). The diameter of the fluid "rope" was approximately 0.1 in, or 0.3 cm. The velocity of the fluid was estimated by using the time for the rope to fall and its height, which were approximately 0.5 seconds and 12 inches (0.3 m), respectively. This yields a velocity of roughly 0.6 m/s. The viscosity of the fluid is a function of both position and time, and therefore difficult to define. Qualitatively, the viscosity of the flowing liquid was on the order of that of honey, which has a kinematic viscosity of 60 cm^2s^{-1} (Maleki, et al 2004). This viscosity will be used to calculate the Reynolds and Froude numbers. Nagahiro et al (2008) used the diameter of the fluid "rope" as the characteristic length when calculating the Froude number.

$$Fr = \frac{V}{\sqrt{gd}} \sim \frac{\left(0.6\frac{m}{s}\right)}{\sqrt{(9.8\frac{m}{s^2} * 0.003m)}} \sim 4$$

$$Re = \frac{dV}{\nu} \sim \frac{\left(0.003m * 0.6\frac{m}{s}\right)}{60\frac{cm^2}{s}} \sim 0.3$$

The Froude number is about 4, which means that the inertial forces are about 4 times more important to the flow than the gravitational forces. The Reynolds number is roughly 0.3, meaning that the viscous forces are about 3 times more relevant than the inertial forces. This suggests that the viscous forces dominate, but inertial forces may also play a significant role. The observed flow is probably a viscous coiling instability, but could also behave as an inertial coiling instability. Using the method proposed by Maleki, et al (2004), the coiling frequency can be estimated by

$$\Omega_\nu = \frac{Q}{Hd^2} = \frac{\left(\frac{\pi}{4}d^2V\right)}{Hd^2} = \frac{\frac{\pi}{4}V}{H} \sim \frac{\left(\frac{\pi}{4} * 0.6\frac{m}{s}\right)}{0.3\ m} \sim 2Hz$$

$$\Omega_I = \left(\frac{Q^4}{\nu d^{10}}\right)^{\frac{1}{3}} = \left(\frac{\left(\frac{\pi}{4} d^2 V\right)^4}{\nu d^{10}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{\left(\frac{\pi}{4} V\right)^4}{\nu d^2}\right)^{\frac{1}{3}} \sim \left(\frac{\left(\frac{\pi}{4} * 0.6 \frac{m}{s}\right)^4}{60 \frac{cm^2}{s} * (0.003m)^2}\right)^{\frac{1}{3}} \sim 100Hz$$

Where Ω_v is the viscous coiling frequency, Ω_I is the inertial coiling frequency, H is the height, ν is the kinematic viscosity, g is the gravitational acceleration, Q is the flow rate, V is the velocity, and d is the diameter of the fluid "rope." These relations predict the coiling frequency of a fluid "rope" coiling on a solid. This is an acceptable approximation in this case, because the standing suspension behaves as a solid on impact. The predicted inertial and viscous coiling frequencies are 2Hz and 100Hz, respectively. The shutter speed was 1/400s, and lower shutter speeds saw some motion blur. More than a couple of degrees of rotation during the exposure would produce blur, so it is clear that 100 Hz coiling frequencies were not observed. 2 Hz corresponds to a 1.8 degree rotation during the exposure, which is consistent with the photograph and the motion blur observed at lower shutter speeds. This suggests that the coiling instability observed can be treated as viscous.

The suspension contained 2⅓ cups corn starch and 1⅓ cups water. This mixture was divided between the pans, with about twice the fluid in the larger pan. Both the red mixture in the small pan and blue mixture in the large pan were dyed using food coloring: 10 drops for the blue and 5 for the red. The red suspension was poured from a height of 12 inches (.3m) into the blue suspension. The photo was taken outside on a porch with walls on the right, front, and back. The ambient temperature was about 60̊ F, but the fluid was mixed inside and not allowed to equilibrate. The sky was cloudy, but there was significant ambient light.

The field of view was about 3" by 2.5" in the final image and 6" x 4.5" in the original. The distance from the lens to the "rope" was around a foot and a half and the lens had a focal length between of 29.2 mm. The camera is an 8 megapixel Canon Powershot A630. The original image was 3264 x 2448 pixels, while the final picture was 1759 x 1342 pixels. The aperture, shutter speed, and ISO were 7.1, 1/400 sec, and 200, respectively. The shutter speed was given priority because slower than 1/400 sec speeds allowed motion blur. ISOs higher than 200 produced grainy images, while lower values required a large aperture. The pan was moved during pouring so that the "rope" would fall into blue fluid, and it would have been difficult to maintain focus with a short depth of field. Because of this, a medium to high F stop was desirable, and 7.1 seemed to provide the necessary depth of field. Using Photoshop, the contrast and brightness were adjusted using the curves function. The original image has somewhat bland colors, but the adjustment largely corrected this problem. Several unsightly bubbles in the blue suspension were removed using the clone stamp. The image was cropped to increase the prominence of the flow in the picture, and frame the point of interest so that it was off-center.

This image shows a viscous coiling instability in a non-Newtonian fluid. The red traces in the blue fluid show the meandering path of the fluid "rope" and its coalescence with the blue suspension. The well defined impact between the falling fluid "rope" and the stationary blue suspension and their somewhat abrupt joining illustrates the shear-thickening behavior of the fluid. The coiling instability is very clearly displayed, while the non-Newtonian nature of the fluid is more difficult to see. The colors in the image weren't quite as bright as I had hoped, and could probably contrast better. I like that the coiling path of the fluid is easy to see, and that the boundary between the "rope" and blue fluid is well defined. Taking a video instead of a picture could provide a much more accurate measurement of the coiling frequency, and injecting the fluid through an orifice at a known speed could provide a better estimation of the Reynolds and Froude numbers. A better estimation of viscosity could also improve the accuracy of these numbers. The best way to further develop this idea is a more rigorous comparison of theoretical and observed coiling frequency.

Works Cited

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