Team Image 1:
High Speed Drop of Water

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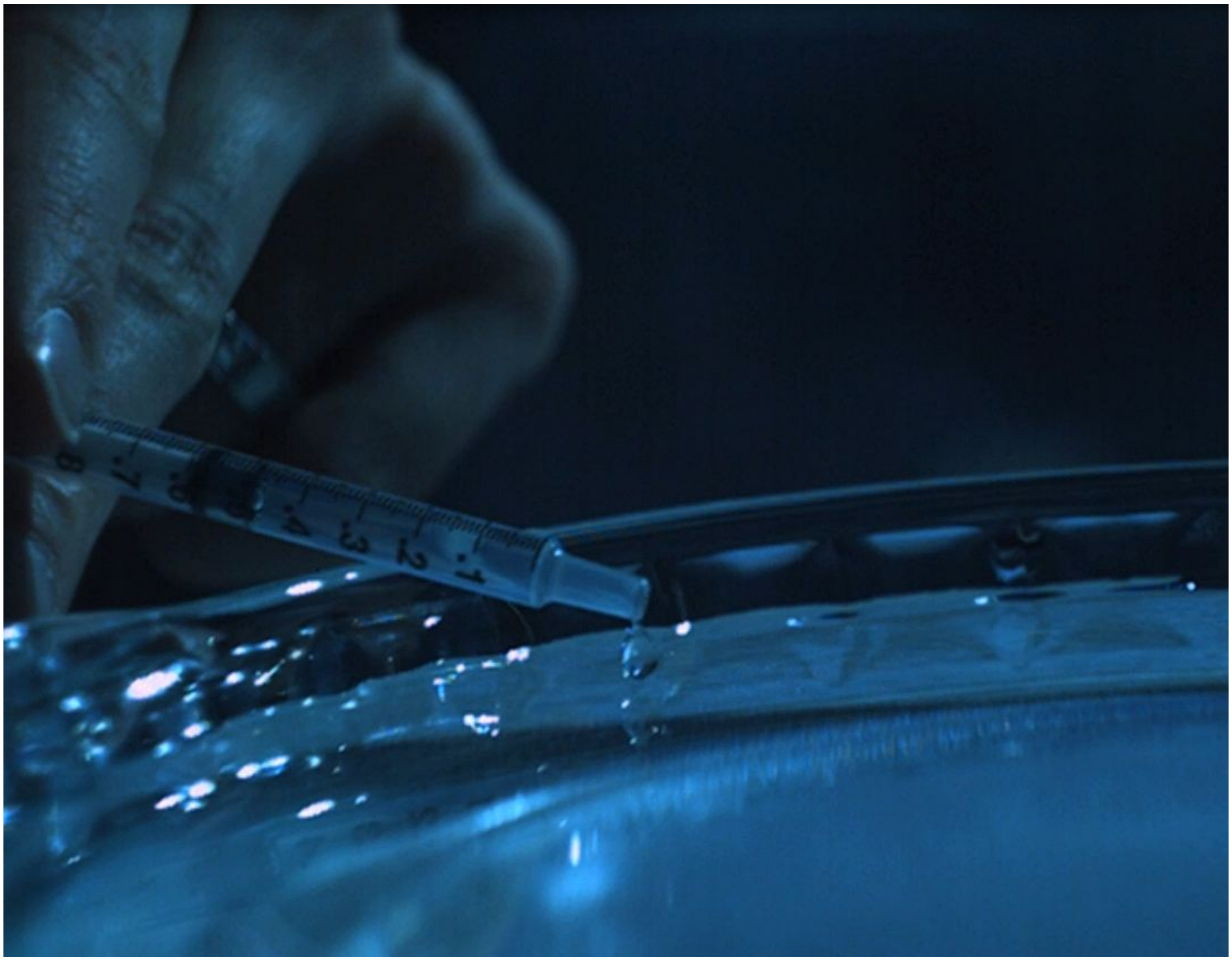
MCEN 5151: Flow Visualization



Date of video: 3/9/2012

Purpose:

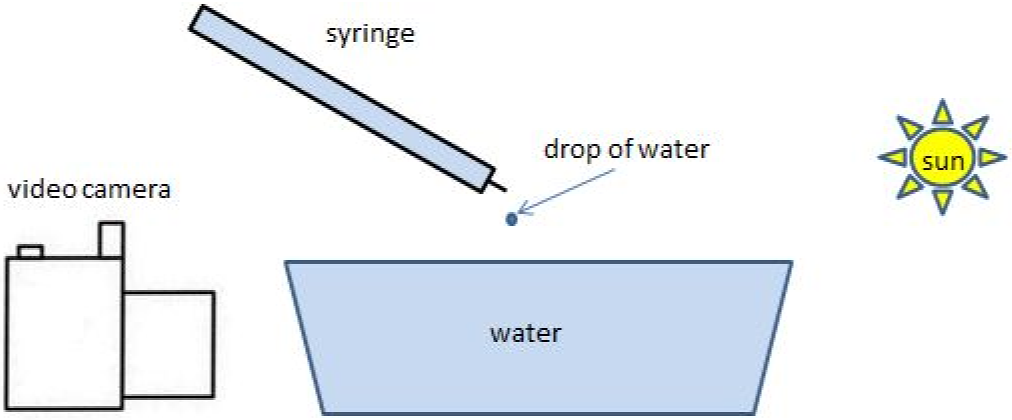
The purpose of this image is to capture the physics of how a single drop of water reacts as it is dropped into a bowl of water from at a small distance from the surface of the water, and low velocity as it impacts the water. The video was captured using a high-speed camera, as it reveals details that are not noticeable when observing the droplet with the human eye.

Apparatus:

The equipment used for this setup included a shallow, glass bowl filled partly with water (about 1.5 inches deep) and a plastic syringe filled with. An Olympus i-SPEED 2 high speed camera was used to film the water droplet falling from the syringe into the bowl of water. The only light source was sunlight, which was quite bright at the time of shooting this video.

Once the high speed camera was filming, I pushed one drop of water out of the plastic syringe and filmed its movement at 1,000 frames per second (fpm) until the water droplet was no longer visible as it had mixed with the water in the bowl.

The camera was held about one foot away from the glass bowl in order to film the image in focus. The total shot lasted less than one second. The following is a visual representation of my apparatus:



Flow Phenomena Observed:

Unfortunately, the phenomenon cannot be seen through the still image included on the cover of this report. However, a simple search on-line for “drop of water floating on water in slow-motion” will give a good idea of the video I was trying to replicate.

Basically, what happens is that the force of gravity causes the water droplet to fall into the body of water from when it detaches from the plastic syringe. After the drop of water impacts the surface of the water in the bowl, a smaller drop bounces into the air and then lands on the surface of water without immediately being absorbed. It simply floats on the surface of the water pool for some time before it again causes another smaller drop of water to bounce into the air or completely gets absorbed into the water pool.

A detailed explanation of the above phenomenon is given by MIT mathematician, John Bush: "There is an air layer between the two, and basically it will coalesce as soon as the drop makes contact with the bath, but it takes a finite amount of time for the air layer to drain."[1]

Simply put, there's a thin layer of air that separates the drop from the surface of the water. However, as the air is pushed aside from the weight of the drop, most of the drop comes in contact with the water below, which causes the drop to become absorbed into the water. This happens so fast that the connection is actually pinched off, which results in the formation of a smaller droplet.

The reason the droplet leaps into the air after the connection is pinched off is, "when it coalesces, waves are generated at the point of contact, and they sweep upwards and they apply a force, which lifts the drops off the surface."[2] Then the water that gets pushed off the surface has enough surface tension to pull itself completely off the water surface and pull itself together into a droplet again. This cycle repeats itself until the droplet is small enough to be completely absorbed by the body of water.

There are various results that can come about from the impact of a liquid droplet with a liquid surface, including floating, bouncing, coalescing, and splashing on the liquid surface.[3] For this project, splashing did not take place.

In order to describe the flow characteristics of the drop, we can calculate a few different dimensionless numbers; namely, the Reynolds, Froude, and Weber numbers. Although I will calculate all of these numbers in this analysis, the impact Weber number is the main parameter that influences the transition of a water drop from coalescing to splashing.[4] The Weber Number is often useful in analyzing fluid flows where there is an interface between two different fluids.[5] The Reynolds number gives a measure of the ratio of inertial forces to viscous forces[6]. The Froude number is a ratio of inertial and gravitational forces.[7]

Based on previously-acquired data from detailed experiments cited in this report (Manzello and Yang), there are certain ranges of values we can expect for the three dimensionless numbers. For example, for Reynolds and Froude numbers greater than 3,000 and between 6 and 18, respectively, the impact of a water droplet with a water pool results in a splash.[8] Also, based on the same experiment, an impact

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[1] http://slowvideos.com/

[2] Ibid.

[3] S. L. Manzello, J. C. Yang, "An experimental study of a water droplet impinging on a liquid surface," *Experiments in Fluids* 32, (2002): 580

[4] Ibid.

[5] http://www.easycalculation.com/physics/fluid-mechanics/weber-number.php

[6] http://www.grc.nasa.gov

[7] http://www.fsl.orst.edu/geowater/FX3/help/8_Hydraulic_Reference/Froude_Number_and_Flow_States.htm

[8] Rodriguez F, Mesler RJ, "Some drops don't splash," *J Colloid Interface Sci*, (1985): 347-352.

Weber number of 5.5 resulted in the water droplet coalescing with the surface of the water pool, and no jet (Worthington jet) formed.[9]

Based on the above statements, I can expect my Reynolds and Froude numbers to be much lower than 3,000 and 6, respectively, and I can also expect a Weber number of slightly higher than 5.5, as my drop didn't completely coalesce, and indeed formed a very subtle jet as it bounced back up. The assumptions will now be verified through calculations. For the sake of simplicity, I will assume that the water droplet is a rigid, spherical object. Based on the still image, the water droplet has a diameter of about 0.19 inches (0.0048 m). The density, dynamic viscosity, and surface tension of water at 25 degrees C are 996.9 kg/m^3, 8.9 E -4 Ns/m^2, and .072 N/m, respectively.[10]

The first calculation to consider is the Reynolds number, which is defined by the following equation:

$$Re = \frac{\rho V D}{\mu} \qquad \text{eq. (1)}$$

Where ρ is the density of the water droplet, V is the velocity of the water droplet, D is the diameter of the water droplet, and μ is the dynamic viscosity of water. All the variables are known except for the velocity, which can be calculated based on a simple kinematic equation of motion with constant acceleration:

$$V_f^2 - V_o^2 = 2a\Delta x \qquad \text{eq. (2)}$$

Where V_f is the final velocity of the droplet (impact velocity), V_0 is its initial velocity, a is the constant acceleration of the water droplet (acceleration due to gravity), and Δx is the displacement, which in this case, is the difference between the initial height of the droplet and the surface of the water pool. The distance traveled is measured from the still image to be about 0.25 inches, or about 0.63 cm. Knowing that the droplet starts at rest and the acceleration due to gravity is approximately 9.8 m/s^2 we have:

$$V_f^2 - V_o^2 = 2a\Delta x \rightarrow V_f = \sqrt{2 * 9.8 * .0063} \rightarrow V_f \approx 0.35\ m/s$$

Knowing the impact velocity of the water droplet (which is about 0.79 mph), the Reynolds number can be calculated:

$$Re = \frac{\rho V D}{\mu} \rightarrow Re = \frac{996.9 * .35 * .0048}{8.9\ E - 4} \rightarrow Re \approx 1880$$

Which is a reasonable, expected value for the Reynolds number, considering there was no splash for the water droplet in my project.

The Froude number is calculated based on the following definition:

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[9] S. L. Manzello, J. C. Yang, "An experimental study of a water droplet impinging on a liquid surface," *Experiments in Fluids* 32, (2002): 588

[10] Ibid. p 581

$$Fr = \frac{V^2}{gD} \qquad \text{eq. (3)}$$

Where g is the acceleration due to gravity. Thus, the Froude number is calculated like so:

$$Fr = \frac{V^2}{gD} \rightarrow Fr = \frac{.35^2}{9.8 * .0048} \rightarrow Fr \approx 2.6$$

This Froude number also agrees with the expected value of less than 6, as again, there is no splash.

Finally, we can calculate the Weber number, which, as stated before, is the most indicative of the transition of a water drop from coalescing to splashing.[11] The Weber number is defined as:

$$We = \frac{\rho V^2 D}{\sigma} \qquad \text{eq. (4)}$$

Where σ is the surface tension of water. The Weber number is calculated to be:

$$We = \frac{\rho V^2 D}{\sigma} \rightarrow We = \frac{996.9 * .35^2 * .0048}{.072} \rightarrow We \approx 8.1$$

Which is also, as expected, slightly greater than 5.5, because the water droplet in my video didn't completely coalesce with the surface of the water pool, as it had a slight upward bounce.

Based on the above calculations of the Reynolds, Froude, and Weber numbers, I have been able to effectively classify the water droplet as a droplet that doesn't cause a splash, but doesn't completely coalesce, so it results in the other two possible forms mentioned by Manzello and Yang, of floating and bouncing, which is exactly what is shown in the video.

Possible Sources of Error:

Although the results obtained in the analysis section are quite reasonable, there are quite a few possible sources of error. First of all, my assumption of treating the water droplet as a solid, spherical object is not entirely accurate, which affects all three dimensionless numbers considered. Also, my measurements for the apparent velocities and diameters of the drop were not very precise, as I was estimating their values based on observing the still image of my video. This would have been much more precise if I had a ruler next to the glass cup as I was taking photos of the fluid motion. Finally, I ignored any water movement caused by previously-dropped water droplets that did not result in the final video I chose for this project. It would have been better if I had waited for the surface of the water pool to reach a state of equilibrium before dropping anymore drops.

Imaging Techniques:

For this video, I used an Olympus i-SPEED 2 high speed camera. The only source of light was sunlight that was coming through the windows at about a 25 degree angle from the horizontal. In order to have

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[11] S. L. Manzello, J. C. Yang, "An experimental study of a water droplet impinging on a liquid surface," *Experiments in Fluids* 32, (2002): 580

ample light for the video, I shot the video at 1,000 fps (any higher would have made the video too dark as there would not have been enough exposure time for each frame). I would have liked to zoom in more but unfortunately the video would become quite out-of-focus when I got any closer to the drop of water with that particular camera. The playback rate for my video is 10 fps. Knowing that the footage of the drop is 21 seconds, I can calculate the time for actual real-time footage with the following calculation:

$$real\ time = \frac{(\#\ of\ frames\ of\ playback)}{(filming\ rate)} = \frac{(time\ of video\ playback) \times (playback\ rate)}{(filming\ rate)} = \frac{21\ s \times 10\ fps}{1000\ fps} = 0.21\ s$$

Based on the above calculation, I filmed 0.21 seconds of real-time footage of the drop of water, which is quite surprising, considering how much can take place in such a short amount of time!

For the final video, the only manipulation I did was that I increased the mid-level colors using Apple ProRes to make the colors more visible. I then used Windows Movie Maker to add the introduction slide, the starting effect, and the credits at the end.

Analysis and Conclusions:

As mentioned earlier, this video reveals the phenomenon that occurs as a drop of water falls into a pool of water at a low velocity. The visualization of a droplet that is bouncing and floating on water was confirmed with the Reynolds, Froude, and Weber dimensionless numbers.

Overall, I am satisfied with the visualization of this phenomenon, but there are a few things I wish I had done differently. First of all, I would have liked to use a high-speed camera that was capable of taking up-close images so that I could have zoomed in on the drop of water. I also would have liked to have more lighting so that the video would still be very bright even at such a high filming rate. Furthermore, I think the droplet would have been more clearly shown and perhaps have a better floating and bouncing effect had I used a hypodermic needle instead of a plastic syringe, which had a relatively large inner diameter compared to that of a hypodermic needle. Finally, it would be helpful to have the needle fixed in a position that would allow the drop of water to be as close as possible to the water surface as it comes out of the needle while still allowing it to actually drop. It was hard to eyeball this as I was taking the video.

The most challenging part of this project was to capture a phenomenon that was not visible to the human eye. There were a few videos I took that didn’t capture the phenomenon and so I was left with no option but to capture the video I wanted simply by trial and error. I am satisfied that my video shows the floating and bouncing phenomenon, but it would have been more clear had I had a more controlled environment and more appropriate tools.